Vibration of Timoshenko beam subjected to partially distributed moving load

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This paper examines the vibration of Timoshenko beam subjected to partially distributed moving load. The governing partial differential equation were analysed to determine the behaviour of the system under consideration. The methods of series solution and numerical method were used to solve the governing equation. Result revealed that the amplitude increases as the fixed length of the beam increases. It was also found that the response amplitude increases as the foundation moduli increases and as the fixed length of the beam increases.

Key words: Timoshenko beam, partially distributed moving load, vibration, amplitude, moving load.

INTRODUCTION

There has always been a description through a system of second order differential equations, in which the vibration amplitude and the angle due to pure bending were the searched functions. Boundary conditions related to the initial-boundary value problem under consideration were described by a proper second order differential equation of both or only one of these functions. However, there is a remark arising from the fact that in the case of the simply supported beam, the boundary conditions are described by the same differential equations as in the case of the Euler-Bernoulli beam (Catal, 2002; Catal, 2006a, b).

In Euler-Bernoulli beam theory, shear deformation and rotation effects are neglected, and plane sections remain plane and normal to the longitudinal axis, while in Timoshenko beam theory, plane sections still remain plane but no longer normal to the longitudinal axis. The Timoshenko model is an extension of the Euler-Bernoulli model by taking into account two additional effects: shearing force and rotatory motion effects.

In this paper, analytical-numerical method was used to determine the response of Timoshenko beam subjected to partially distributed moving load.

Definition 1

Beam is a piece of horizontal or vertical structure that is capable of withstanding load primarily by resisting bending. The bending force is introduced into the material of the beam as a result of external load, own weight, span and external reaction (Chang, 2000; Chen, 1963).

Beams are traditionally description of building or civil engineering structural elements, but smaller structures such as truck or automobiles frames, machine frames and other mechanical structural systems contains beam structures that are designed and analysed in a similar fashion (Chree, 1889; Dowell, 1979). There are three basic types of beam which include:

(i) Simple span, supported at both ends.
(ii) Continuous, supported at more than two points.
(iii) Cantilever, supported at one end with the other end overhanging and free.

It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam (Doyle and Pavlovic, 1982; Farghaly, 1994; Gurgoze, 1984; Gurgoze, 1996). Introducing the mass by the Dirac delta function, Chen (1963) solved analytically the problem of a Simply Supported beam carrying a concentrated mass. Cha (2001) obtained the natural frequencies of a continuous structure with spring mass attachments by using the classical assumed-modes method in conjunction with Langrange's equation.
THE GOVERNING EQUATION

The governing equations describing the vibration behaviour of a Timoshenko beam subjected to partially distributed moving load are (Catal, 2002):

\[ EI \frac{\partial^4 w(x,t)}{\partial t^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} + Kw(x,t) = P_f(x,t) \]  

(1)

where \( E \) is Young's modulus, \( I \) is the constant moment of inertia of the beam's cross section about the axis, \( A \) is the area, \( w(x,t) \) is the deflection of the beam, \( K \) is the Winkler foundation, \( \rho \) is the density, \( t \) is the time, \( x \) is the spatial coordinate and \( P_f(x,t) \) is the applied force (i.e. the resultant concentrated force caused by the moving mass).

The applied force per unit length \( P_f(x,t) \) is the uniform partially distributed moving load which is defined as:

\[ P_f(x,t) = \frac{1}{\epsilon} \left[ -Mg - M \frac{\partial^2 w}{\partial x^2} \right] \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \]  

(2)

Where \( M \) is the mass of the load, \( g \) is the acceleration due to gravity, \( \xi \) is the fixed length of load, \( \epsilon \) is the length of the beam. The differential operator \( \frac{\partial^2 w(x,t)}{\partial x^2} \) is defined as:

\[ \frac{d^2 w(x,t)}{dx^2} = \frac{\partial^2 w(x,t)}{\partial t^2} + 2V \frac{\partial^2 w(x,t)}{\partial x \partial t} + V^2 \frac{\partial^2 w(x,t)}{\partial x^2} \]  

(3)

\( H \) is the heavy-side function such that:

\[ H(x - \xi + \frac{\epsilon}{2}) = H(x - (\xi - \frac{\epsilon}{2})) = \begin{cases} 0 & x < \xi - \frac{\epsilon}{2} \\ 1 & x > \xi - \frac{\epsilon}{2} \end{cases} \]

\[ H(x - \xi - \frac{\epsilon}{2}) = H(x - (\xi + \frac{\epsilon}{2})) = \begin{cases} 0 & x < \xi + \frac{\epsilon}{2} \\ 1 & x > \xi + \frac{\epsilon}{2} \end{cases} \]  

(4)

Hence, the governing equation becomes:

\[ EI \frac{\partial^4 w(x,t)}{\partial t^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} + Kw(x,t) = \frac{1}{\epsilon} \left[ -Mg - M \frac{\partial^2 w(x,t)}{\partial x^2} + 2MV \frac{\partial^2 w(x,t)}{\partial x \partial t} + V^2 \frac{\partial^2 w(x,t)}{\partial x^2} \right] \]  

(5)

With the boundary conditions:

\[ w(0,t) = 0 = w(L,t) \]  

(6)

\[ \frac{\partial^2 w(0,t)}{\partial x^2} = 0 = \frac{\partial^2 w(L,t)}{\partial x^2} \]  

(7)

Without loss of generality, one can consider the initial conditions of the form:

\[ w(x,0) = 0 = \frac{\partial w(x,0)}{\partial t} \]  

(8)

METHOD OF SOLUTION

In this section, we proceed to solve the boundary-initial value problems comprising of Equations 1-8. The transverse displacement and external applied force may be expressed as:

\[ w(x,t) = \sum_{i=1}^{\infty} X_i(x) \gamma_i(t) \]  

(9)

Substituting Equation 9 into Equation 5, we have

\[ \frac{EI}{\epsilon} \sum_{i=1}^{\infty} X''_i(x) \gamma_i(t) + \rho A \sum_{i=1}^{\infty} X_i(x) \gamma''_i(t) + K \sum_{i=1}^{\infty} X_i(x) \gamma_i(t) \]

\[ = \left[ \frac{Mg}{\epsilon} \sum_{i=1}^{\infty} \gamma_i(t) X_i(x) - \frac{2MV}{\epsilon} \sum_{i=1}^{\infty} \gamma'_i(t) X'_i(x) - \frac{V^2M}{\epsilon} \sum_{i=1}^{\infty} \gamma''_i(t) X''_i(x) \right] \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \]  

(10)

Furthermore, we assume that:

\[ P_{ff}(x,t) = \sum_{i=1}^{\infty} \gamma_{fi}(t) X_i(x) \]  

(11)

Substituting Equation 11 into Equation 10, we have:

\[ = \sum_{i=1}^{\infty} \gamma_{fi}(t) X_i(x) - \frac{Mg}{\epsilon} \left( H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right) - \frac{2MV}{\epsilon} \sum_{i=1}^{\infty} \gamma'_i(t) X'_i(x) \]

\[ \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] - \frac{V^2M}{\epsilon} \sum_{i=1}^{\infty} \gamma''_i(t) X''_i(x) \]  

(12)

To normalize Equation 12, we multiply all through by \( X_j(x) \) to obtain:

\[ \sum_{i=1}^{\infty} \gamma_{ji}(t) X_i(x) X_j(x) = \frac{Mg}{\epsilon} \left( H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right) \]

\[ - \frac{M}{\epsilon} \sum_{i=1}^{\infty} \gamma''_i(t) X_i(x) X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \]

\[ - \frac{2MV}{\epsilon} \sum_{i=1}^{\infty} \gamma'_i(t) X'_i(x) X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \]

\[ - \frac{V^2M}{\epsilon} \sum_{i=1}^{\infty} \gamma''_i(t) X''_i(x) X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \]  

(13)

Integrating both sides of Equation 13 with respect to \( x \) along the length \( L \) of the beam, we have:

\[ \sum_{i=1}^{\infty} \gamma_{ji}(t) \int_0^L X_i(x) X_j(x) \, dx = \frac{Mg}{\epsilon} \int_0^L X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \, dx \]

\[ - \frac{M}{\epsilon} \sum_{i=1}^{\infty} \gamma''_i(t) \int_0^L X_i(x) X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \, dx \]

\[ - \frac{2MV}{\epsilon} \sum_{i=1}^{\infty} \gamma'_i(t) \int_0^L X'_i(x) X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \, dx \]

\[ - \frac{V^2M}{\epsilon} \sum_{i=1}^{\infty} \gamma''_i(t) \int_0^L X''_i(x) X_j(x) \left[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right] \, dx \]  

(14)
From Equation 14, we assume the following:

\[ 0 = -\frac{Mg}{\epsilon} \int_0^L \int_0^L X_j(x) \left\{ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \right\} dx \]  

(15)

Integrating by part using:

\[ H \left( x - \xi + \frac{\epsilon}{2} \right) - H \left( x - \xi - \frac{\epsilon}{2} \right) \int_0^L X_j(x) \,dx \]

\[ - \int_0^L \int_0^L X_j(x) \left[ H' \left( x - \xi + \frac{\epsilon}{2} \right) - H' \left( x - \xi - \frac{\epsilon}{2} \right) \right] \,dx \,dx \]

(16)

Since \( H'(x) = \delta(x) \),

\[ F(x) = H(x) \quad \text{where} \quad H'(x) - H(x) = \delta(x) \]

(17)

such that we get,

\[ A_{nj} = \frac{Mg}{\epsilon} \int_0^L X_j(\xi + \frac{\epsilon}{2}) \,d\xi - \int_0^L X_j(\xi - \frac{\epsilon}{2}) \,d\theta(x) \]

(18)

Furthermore, expanding using Taylor series, we obtain,

\[ X_j \left( \xi + \frac{\epsilon}{2} \right) = X_j(\xi) + \frac{1}{11} X_j''(\xi) \right] + \frac{1}{21} X_j'''(\xi) + \frac{1}{31} X_j'''''(\xi) + \cdots \]

(19)

Also,

\[ X_j \left( \xi - \frac{\epsilon}{2} \right) = X_j(\xi) - \frac{1}{11} X_j''(\xi) \right] + \frac{1}{21} X_j'''(\xi) - \frac{1}{31} X_j'''''(\xi) + \cdots \]

(20)

By substituting Equations 19-20 into Equation 18, we have:

\[ A_{nj} = \frac{Mg}{\epsilon} \int_0^L \left[ X_j(\xi) + \frac{1}{11} X_j''(\xi) + \frac{1}{21} X_j'''(\xi) + \frac{1}{31} X_j'''''(\xi) - X_j(\xi) - \frac{1}{11} X_j''(\xi) \right] \frac{\epsilon^3}{24} X_j'''(\xi) + \frac{\epsilon^3}{24} X_j'''(\xi) \]

(21)

We get,

\[ \epsilon X_j'(\xi) + \frac{\epsilon^3}{24} X_j'''(\xi) \]

(22)

Substituting Equation 22 into Equation 16 and having satisfied the condition (4), we have,

\[ 0 = -Mg \left[ X_j(\xi) + \frac{\epsilon^2}{24} X_j''(\xi) \right] \]

(23)

Similar arguments is applicable to second, third and fourth definite integral in Equation 14, hence, evaluating the integrals using Taylor's series expansion and applying orthogonality properties of the characteristics function \( \gamma_{j}(t) \) the left hand side of Equation 14, we finally obtain:

\[ \gamma_j(t) = -Mg \left[ X_j(\xi) + \frac{\epsilon^2}{24} X_j''(\xi) \right] \]

\[ -\gamma \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''(\xi) + 2X_i''(\xi) X_j(\xi) + 2X_i''(\xi) X_j''(\xi) \right) \right] \]

\[ -2\gamma \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''(\xi) + 2X_i''''(\xi) X_j(\xi) + 2X_i''''(\xi) X_j''(\xi) \right) \right] \]

\[ -V^2 \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''''(\xi) + 2X_i''''''(\xi) X_j(\xi) + 2X_i''''''(\xi) X_j''(\xi) \right) \right] \]

(24)

Furthermore, from Equation 10 we have that:

\[ E \sum_{i=1}^{N} X_i''(\xi) \gamma_i(t) + \rho A \sum_{i=1}^{N} \gamma_i(t) \gamma_i(t) + k \sum_{i=1}^{N} \gamma_i(t) \gamma_i(t) = \sum_{i=1}^{N} \gamma_i(t) X_i(\xi) \]

(25)

Substituting Equation 24 into Equation 25 becomes:

\[ E \sum_{i=1}^{N} X_i''(\xi) \gamma_i(t) + \rho A \sum_{i=1}^{N} \gamma_i(t) \gamma_i(t) + k \sum_{i=1}^{N} \gamma_i(t) \gamma_i(t) = -Mg \left[ X_j(\xi) + \frac{\epsilon^2}{24} X_j''(\xi) \right] \]

\[ -\gamma \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''(\xi) + 2X_i''(\xi) X_j(\xi) + 2X_i''(\xi) X_j''(\xi) \right) \right] \]

\[ -2\gamma \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''(\xi) + 2X_i''''(\xi) X_j(\xi) + 2X_i''''(\xi) X_j''(\xi) \right) \right] \]

\[ -V^2 \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''''(\xi) + 2X_i''''''(\xi) X_j(\xi) + 2X_i''''''(\xi) X_j''(\xi) \right) \right] \]

(26)

so that Equation 26 becomes:

\[ E \sum_{i=1}^{N} X_i''(\xi) \gamma_i(t) + \rho A \sum_{i=1}^{N} \gamma_i(t) \gamma_i(t) + k \sum_{i=1}^{N} \gamma_i(t) \gamma_i(t) = -Mg \left[ X_j(\xi) + \frac{\epsilon^2}{24} X_j''(\xi) \right] + M \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''(\xi) + 2X_i''(\xi) X_j(\xi) + 2X_i''(\xi) X_j''(\xi) \right) \right] + 2\gamma \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''(\xi) + 2X_i''''(\xi) X_j(\xi) + 2X_i''''(\xi) X_j''(\xi) \right) \right] + V^2 \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''''(\xi) + 2X_i''''''(\xi) X_j(\xi) + 2X_i''''''(\xi) X_j''(\xi) \right) \right] = 0 \]

(27)

Next, we consider the free vibration of a Timoshenko beam under consideration, thus we have:

\[ X_i''(x) - \beta_i^4 \alpha(t) = 0 \]

(28)

\[ \alpha_i^2 = \frac{\beta_i^4 EI}{M} \]

(29)

Where \( \alpha_i^2 \) is the circular frequency of the beam. Considering Equations 28 and 29, Equation 27 yields:

\[ \sum_{i=1}^{N} X_i'' \left[ M \alpha_i^2 \gamma_i + \rho A \beta_i^4 \gamma_i + k \gamma_i \right] + M \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''(\xi) + 2X_i''(\xi) X_j(\xi) + 2X_i''(\xi) X_j''(\xi) \right) \right] \]

\[ + 2\gamma \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''(\xi) + 2X_i''''(\xi) X_j(\xi) + 2X_i''''(\xi) X_j''(\xi) \right) \right] + V^2 \sum_{i=1}^{N} \gamma_{i}(t) \left[ X_i(\xi) X_j''''(\xi) + \frac{\epsilon^2}{24} \left( X_i(\xi) X_j''''''(\xi) + 2X_i''''''(\xi) X_j(\xi) + 2X_i''''''(\xi) X_j''(\xi) \right) \right] = 0 \]

(30)

Equation 30 above must be satisfied for arbitrary \( X_i(x) \), and this is possible only when the expression in the curly bracket is equal to zero. We therefore obtain:
For the boundary conditions given under the governing equation:

\[ X_i(x) = \sqrt{\frac{2}{L}} \sin \left[ \frac{\pi x}{L} \right] \]  

(32)

To obtain a set of exact governing differential equation for the Simply Supported beam under consideration, we substitute Equation 32 into Equation 14 to obtain:

\[ \sum_{i=1}^{n} \gamma_i(t) \int_{0}^{L} \sin \frac{\pi x}{L} \sin \left[ \frac{\pi x}{L} \right] dx = -M_0 \sum_{j=1}^{n} \gamma_j(t) \int_{0}^{L} \sin \frac{\pi x}{L} \sin \left[ \frac{\pi x}{L} \right] dx - 2M \sum_{i=1}^{n} \gamma_i(t) \int_{0}^{L} \sin \frac{\pi x}{L} \sin \left[ \frac{\pi x}{L} \right] dx - 2M_0 \gamma_0(t) \int_{0}^{L} \sin \frac{\pi x}{L} \sin \left[ \frac{\pi x}{L} \right] dx \]  

(33)

Evaluating the above integrals, we have:

\[ Q_1 = \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] H\left( x - \frac{\xi}{2} \right) - H\left( x - \frac{\xi}{2} \right) dx = 2 \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] \]  

(34)

\[ Q_2 = \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] H\left( x - \frac{\xi}{2} \right) - H\left( x - \frac{\xi}{2} \right) dx \]  

\[ = \frac{1}{n} \cos \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \]  

(35)

\[ Q_3 = \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] H\left( x - \frac{\xi}{2} \right) - H\left( x - \frac{\xi}{2} \right) dx \]  

\[ = \frac{1}{n} \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \]  

(36)

\[ Q_4 = \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] H\left( x - \frac{\xi}{2} \right) - H\left( x - \frac{\xi}{2} \right) dx \]  

\[ = \frac{1}{n} \cos \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi x}{L} \right) \]  

(37)

\[ Q_5 = \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \]  

(38)

By substituting equations Equation 34- 38 into Equation 33, we have:

\[ \gamma_i(t) = -M_0 \sum_{j=1}^{n} \gamma_j(t) \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx + 2M \sum_{i=1}^{n} \gamma_i(t) \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx + 2M_0 \gamma_0(t) \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx \]  

(39)

By replacing the right hand side of Equation 31 with the right hand side of Equation 39, we finally obtain:

\[ M_0 \gamma_i(t) + \rho A \gamma_i(t) + K_0 \gamma_i(t) = -M_0 \frac{\pi \xi}{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] + \pi \xi \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx \]  

(40)

\[ -4M \sum_{i=1}^{n} \gamma_i(t) \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx - 4M \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx \]  

\[ -2M_0 \frac{\pi \xi}{L} \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx - 2M_0 \gamma_0(t) \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx \]  

(41)

\[ \gamma_i(t) = (\gamma_{i+1} - 2\gamma_i + \gamma_{i-1}) h^2 \]  

(42)

where,

\[ \gamma_i(t) = (\gamma_{i+1} - \gamma_{i-1}) / 2h \]  

(43)

\[ \rho A \left( \gamma_{i+1} - 2\gamma_i + \gamma_{i-1} \right) + M (\sigma(t) + K) \gamma_i = -M_0 \frac{\pi \xi}{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] + \pi \xi \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx \]  

(44)

Multiplying through by we finally obtain, \( h^2 \)

\[ \left\{ \begin{align*} &\rho A \left( \gamma_{i+1} - 2\gamma_i + \gamma_{i-1} \right) + M (\sigma(t) + K) \gamma_i = -M_0 \frac{\pi \xi}{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] + \pi \xi \int_{0}^{L} \sin \left[ \frac{\pi x}{L} \right] \sin \left[ \frac{\pi x}{L} \right] dx \end{align*} \right\} \]  

(45)
Figure 1. Deflection of beam for $\rho A = 0.02$ at different values of $\xi$.

Figure 2. Deflection of beam for $\rho A = 0.04$ at different values of $\xi$.

NUMERICAL SIMULATION

Computer program (MATLAB) was developed and the following numerical data were used:

$\rho A = 0.2, 0.4, 0.6, 0.8, 1.0 \text{ kg/m}; \xi$ and $\zeta = 0.01, 0.02$ and $0.03$; $M = 70 \text{ kg}, L = 6 \text{ m}, m = 7 \text{ kg}, v = 10 \text{ m/s}, K = 0, 1, 2, 3$ and 4.

Hence we have the graphs showing the results (Figures 1 – 4). The deflection profiles of the beam are displayed graphically to demonstrate the effect of fixed length of the
beam, fixed length of the load and the foundation constant.

**Conclusion**

From the response profile of the beam, it was observed that irrespective of the value of the damping coefficient, the response amplitude of vibration increases as the fixed length of the load increases. It was also found that the response amplitude of vibration increases as the foundation moduli increases at fixed length of the beam.

**REFERENCES**


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